

3D Model of the Winding Product

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Abstract

The article presents the results of computer modeling of a winding product. It describes the method for creating a 3D model based on a series of flat sections passing through the axis of the package. The issues encountered during 3D modeling and their solutions are discussed. The authors propose using the continued fraction method to describe the structure of winding products and optimize the model for subsequent calculations in CAE programs. The article provides information about the process of creating a 3D model of the winding product, as well as the possibilities for reducing computational resources required for modeling. The results of computer modeling of various winding types are presented. In the conclusion, the authors assert that the developed method for analyzing and synthesizing the structure of precision winding using the continued fraction approach allows for the exploration of various winding types and the creation of optimal 3D models that replicate their structure. These models can be used for further research and as digital models of winding products.

Keywords: three-dimensional winding model, computer simulation, winding product, digital model, winding types, virtual twin, winding production, winding calculation, winding model calculation, experimental sample.

1. Introduction

Winding products (WP) represent either a finished product or a workpiece used for the creation of composite materials, electrical coils of various shapes and purposes, and filtering elements of the winding type. In such products, layers of wound threads of different natures, ranging from volumetric textured threads to wire, form filtering barriers for the retention of mechanical particles, liquids, and gases. Depending on the application, materials with the required mechanical, thermal, and chemical properties are used for winding, ensuring durability and high efficiency of the products in aggressive or extreme operating conditions.

Numerous studies have been devoted to the application and development of fiber-reinforced polymer composite materials (PCM) in various industries. In the petrochemical industry, pipes based on PCM are widely used [1]. In aerospace and rocket production, the winding method is used to create aircraft fuselages and airplane wings [2, 3, 4, 5, 6], among other components. Winding products are also employed in shipbuilding, as evidenced by the works [7, 8]. In the chemical industry, PCM-based pipes with specific hydraulic and mechanical properties are produced [9, 10]. Lightweight and strong materials are in demand in mechanical engineering, and studies in this area are presented in works [11, 12].

Winding products, used as filtering aerators, are discussed in the studies [13, 14, 15].

2. Problem Statement

Traditional methods of analysis and design for winding products are limited in their capabilities. The complex internal structure of the winding makes it difficult to analyze its charac-

teristics and optimize production processes. The lack of effective visualization and analysis tools for the structure of winding products complicates the design process, leading to high costs and extended development time. One of the areas for studying the properties of winding products is the development of their computer representation in the form of 3D models. Using this approach allows the analysis of the internal structure of winding products (WP) at the design stage, determination of optimal winding process parameters, and solving optimization tasks related to the structure of WP to achieve the required physical and mechanical properties (strength, stiffness, permeability of the product).

The developed digital models can be widely applied in the creation of composite materials, setting properties for filtering components, determining the quality of yarn dyeing in packages, and other areas of application.

3. Theory

The proposed method for constructing a 3D model of a winding product (WP) includes building multiple cross-sections of the winding product, sequentially connecting the winding turns within the WP body, and assigning volume to the winding turns using 3D modeling tools. The calculation scheme of a cylindrical winding product is shown in Figure 1, where 1 is the starting point of the winding turns, 2 are the sections of the mandrel, 3 are the sections of the threads, and D_H is the outer diameter of the mandrel. A cylindrical coordinate system $OZY\varphi$ is introduced for the modeling, with the OZ axis coinciding with the rotation axis of the WP. The sections of the WP body are made with half-planes α_k passing through the OZ axis. The position of each half-plane relative to the axis is defined by the angle φ_k , measured from a selected initial half-plane in a plane perpendicular to the OZ axis.

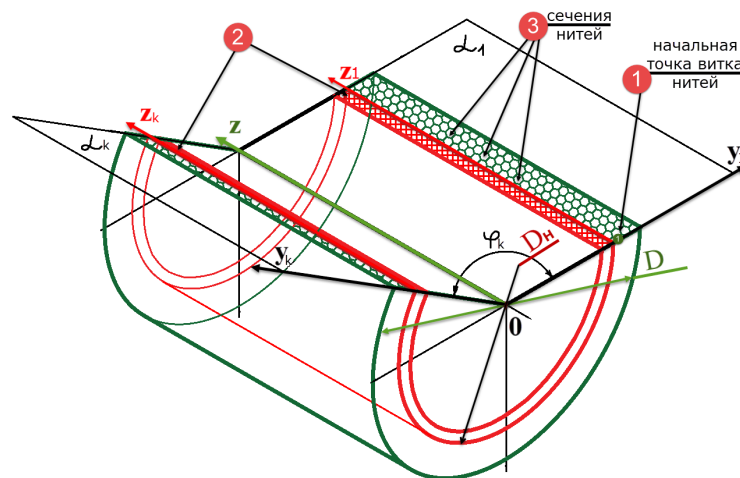


Figure 1: Calculation Scheme of WP Sections by Half-Planes

Let us consider the section formed by plane α_1 (Figure 1). We assume that the winding turns start from the point with coordinates $(0, D_H + 0.5, 0)$ in the cylindrical coordinate system, where d is the thread diameter. The coordinates of the centers of the winding turns in the cross-section of WP by plane α_1 are denoted as z_m^1, z_m^1 , where m is the number of the turn and 1 is the section number. Similarly, we denote the coordinates of the winding turns in corresponding sections of the WP body by half-planes α_k as z_m^k, y_m^k .

When calculating the coordinates of the winding centers, the following assumptions are made: the thread is wound onto the surface of the product in the form of a spiral with a constant pitch; the change in winding direction occurs instantaneously (edge effects are not considered); after laying, the thread does not shift; deformations of the thread and product are considered insignificant and not accounted for; the cross-sections of the threads have a constant diameter and are represented as circles.

During the creation of a computer 3D model of the winding product, the coordinates z_m^k , y_m^k of the centers of the winding turns with the same numbers m are sequentially connected, forming a data array for constructing a 3D spiral. Each new spiral with number m is attached to the previous spiral with number $m-1$, forming a complete set of coordinate points on the central axis of the thread wound onto the mandrel. After this, using the 3D modeling tools of CAD software, the thread in the model gains volume by moving the thread section with diameter d along the trajectory of the 3D spiral consisting of the turns.

The method of constructing the winding turns' sections is described in the paper [16]. The array of center coordinates is calculated in a custom program in MATLAB and saved in a file in EXCEL format. To construct the 3D spiral in KOMPAS-3D, the prepared array of point coordinates is read from the EXCEL file and then transformed into a spline. It should be noted that, as a result of calculations, a large array of point coordinates is generated, which leads to significant time consumption when converting the data into a spline.

After building the 3D spiral and creating the thread cross-section sketch in the form of a circle with diameter d in the selected plane in KOMPAS-3D, the volume is added to the thread along the trajectory of its turns using the "Element along trajectory" operation, which completes the construction of the computer 3D model of the WP. During the construction process, an error "self-intersecting surface" sometimes occurs, caused by the sharp transition of the thread at the ends of the WP (see Figure 2). To eliminate this error, the thread diameter d can be reduced, for example. Figure 3 shows an enlarged fragment of the 3D model of the WP. The mandrel is shown in yellow, over which the thread sections are placed. The larger diameter circles demonstrate the initially specified thread diameter.

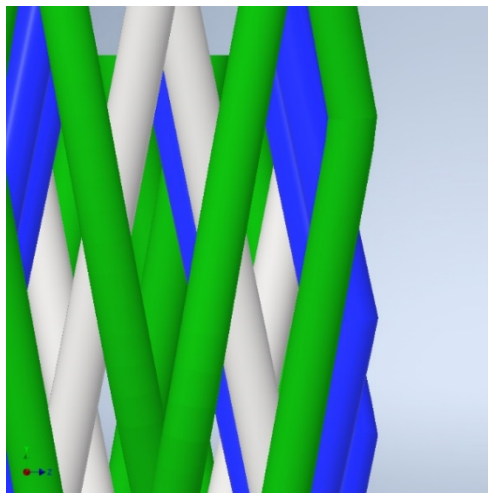


Figure 2 Threads on the edges of the winding product

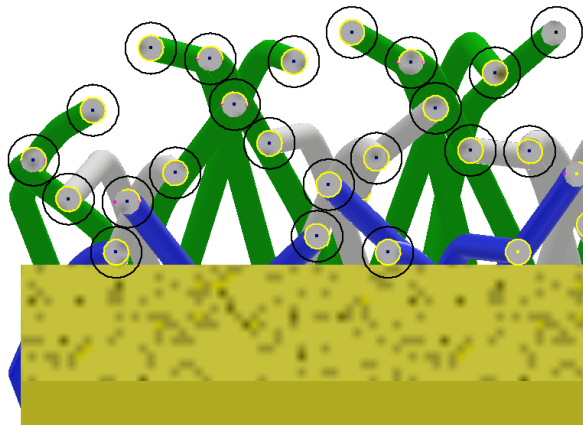


Figure 3 Enlarged cross-sectional fragment of the winding

Using the proposed method, a series of computer 3D models of cylindrical winding products (WP) was constructed. During the construction process of the 3D models, it was revealed that generating a large number of turns simultaneously when creating the 3D spiral leads to significant time consumption, both for converting the data array into a spline and for performing the "Element along trajectory" operation. To reduce these time costs, it is recommended to divide the initial point coordinate data array into several parts. After generating all the parts sequentially, the complete computer 3D model of the WP can be obtained through a merging operation. When the distance between the center points of the sections forming the winding turns is small, the interpolation of the 3D spiral by a spline can be replaced by linear interpolation.

Figure 4 shows the result of obtaining the computer 3D model of the WP for the following parameter values: $B = 60$ mm, $d = 0.5$ mm, $H = 20.65$ mm, $m = 120$.

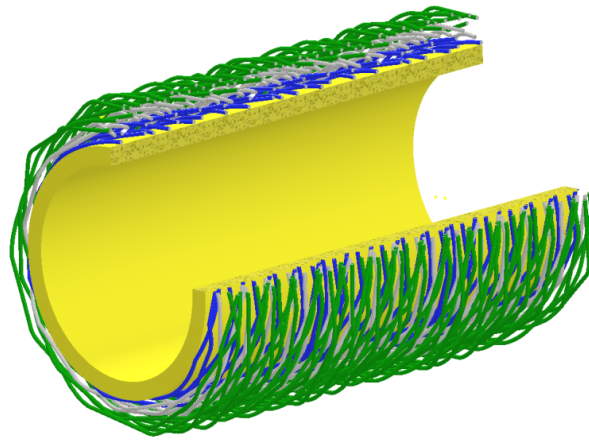


Figure 4 Computer 3D model of the winding product (WP)

From Figure 4, it can be seen that the model is quite complex and contains a large volume of data, which creates limitations for its further application in CAE (computer-aided engineering) programs. Based on this, the model needs to be simplified.

The article [17] shows that the structure of the winding products (WP) can be effectively described using the apparatus of continued fractions.

The initial data for synthesizing the forging includes:

- the diameter d of the wound thread with a circular cross-section;
- the dimensions of the wound WP (B – width of the WP, mm; D – current diameter of the WP, mm);
- the limits within which the angle of the winding threads α must lie, degrees.

As shown in the article [17], the structure of the WP can be described by a sequence of integers $\alpha_0, \alpha_1, \dots, \alpha_n$.

The number α_0 characterizes the pitch of the wound thread spiral H , and together with the values B and D defines the angle of the winding threads, such that $\alpha_0 < \frac{2B}{H} < \alpha_0 + 1$.

To ensure the required winding structure with specified step sizes of the corresponding orders, one can represent k as a continued fraction:

$$k = \frac{2 * B}{H} = \frac{500}{124} = 4 + \frac{4}{124} = 4 + \frac{1}{31} \quad (1)$$

$\alpha_0=4; \alpha_1=31; \alpha_2=\alpha_3=\dots=0,$

and k represents the gear ratio.

Assume we want to obtain winding with the following parameters:

Height of winding = 250 mm,

$H \sim 124 \text{ mm}$, $H_1 \sim 1 \text{ mm}$, тогда $a_0 = E \left(\frac{500}{124} \right) = 4$, and formula (1) for the specified type of winding takes the form $k = 4 + \frac{1}{124} = \frac{497}{124}$

In 497 turns, the thread guide will make 124 double runs. Let's find H.

$$H = \frac{124}{497} * 500 = 124.7485 \text{ mm};$$

Then H_1 can be found using the formulas:

$$H_1 = \min \left\{ \begin{array}{l} 2B - H * E \left(\frac{2 * B}{H} \right) \\ H * \left[E \left(\frac{2 * B}{H} \right) + 1 \right] - 2 * B \end{array} \right. \quad (2)$$

$$H_1 = \min \left\{ \begin{array}{l} 2 * 250 - 124.7485 * E \left(\frac{2 * 250}{124.7485} \right) = 1.0060 \text{ mm} \\ 124.7485 * \left[E \left(\frac{2 * 250}{124.7485} \right) + 1 \right] - 2 * 250 = 123.7425 \text{ mm} \end{array} \right.$$

As a result of the calculations, the following parameters were obtained:

Gear ratio $\frac{497}{124}$: $B=250 \text{ mm}$; $H=124,7485 \text{ mm}$; $H_1=1,0060 \text{ mm}$; $H_2=0 \text{ mm}$ (see Table 1).

Based on formula (1), it is evident that the structure of the forging has repeating elements. To save computer resources, we will keep 2 steps of winding and find new parameters for the gear ratios and the height of winding while maintaining the structure of the winding (H and H_1).

$$k = 2 + \frac{1}{124} = \frac{249}{124}$$

$$H = \frac{124}{249} * (2 * B - H * (4 - 2)) = \frac{124}{249} * (500 - 124.7485 * 2) = 124.7485 \text{ mm}$$

The winding height for two steps will have the following formula:

$$B' = (2B - H * (E \left(\frac{2 * B}{H} \right) - 2)) / 2 \quad (3)$$

From the obtained expression, we can determine B'

$$B' = \frac{500 - 124.7485 * 2}{2} = 125.2515 \text{ mm}$$

Based on the acquired data, all calculations can be performed for other winding products, and computer models of various structures can be constructed, selecting the necessary parameters for their implementation on winding devices (see Table 1). For better visualization, models with two winding steps have been built.

This sequence represents the coefficients of the continued fraction obtained from the decomposition of the rational fraction $k = \frac{2B}{H}$ for example, using the Euclidean algorithm.

$$k = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots}}}$$

Let's consider the repeating turns of the thread in the unfolded view (Figure 5).

The triangles ABC and MDE are isosceles. The heights BH and DK divide the base of the triangles into equal parts. Therefore, $AH + LM = HC + CL$, which means that $HL = H/2$, where H is the turn step.

Based on the graphical solution, it can be observed that the winding steps repeat. Thus, we can isolate an element without disrupting the structure of the winding product and create a model based on it for further calculations in CAE

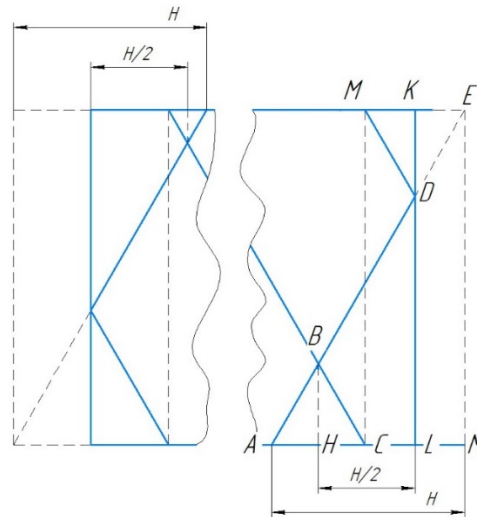



Figure 5 Unfolded view of the thread winding

4. Experimental Results

TABLE 1. WINDING PRODUCTS

<p>Closed structure</p> $u = \frac{497}{124} = 4 + \frac{1}{124};$ <p>B = 250 mm</p> <p>D = 95 mm</p> <p>H = 124.7485 mm</p> <p>H₁ = 1.0060 mm</p> <p>H₂ = 0</p> <p>A = 0.4155</p> <p>Thread diameter d = 0.85 mm</p>	
<p>Same structure, but with different gear ratios</p> $u = \frac{249}{124} = 2 + \frac{1}{124};$ <p>B' = 125.2516 mm</p> <p>D = 95 mm</p> <p>H = 124.7485 mm</p> <p>H₁ = 1.0060 mm</p> <p>H₂ = 0</p> <p>α = 0.4155</p> <p>Thread diameter d = 0.85 mm</p>	<p>Figure 6 Winding product with a closed winding structure, gear ratio $u = \frac{497}{124}$</p>

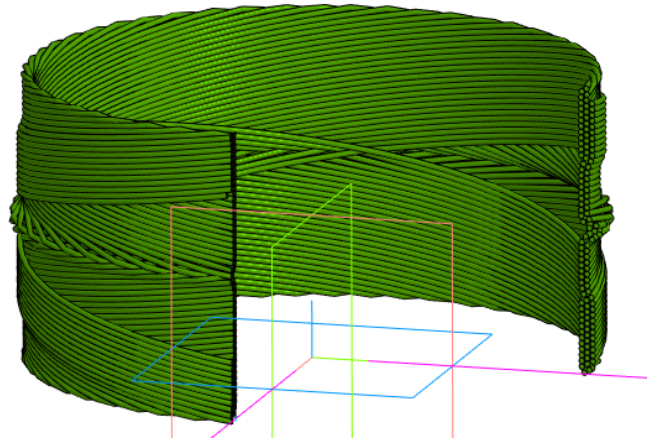


Figure 7 Computer model of thread winding with a closed structure, gear ratio $u = \frac{249}{124}$

Closed structure

$$u = \frac{491}{49} = 10 + \frac{1}{49} = 9 + \frac{99}{49};$$

$B = 250 \text{ mm}$

$D = 95 \text{ mm}$

$H = 49.8982 \text{ mm}$

$H_1 = 1.0183 \text{ mm}$

$H_2 = 0$

$\alpha = 0.1747$

Thread diameter $d = 0.9 \text{ mm}$

Same structure, but with different gear ratios

$$u = \frac{99}{49} = 2 + \frac{1}{49};$$

$B' = 50.4073 \text{ mm}$

$D = 95 \text{ mm}$

$H = 49.8982 \text{ mm}$

$H_1 = 1.0183 \text{ mm}$

$H_2 = 0$

$\alpha = 0.1747$

Thread diameter $d = 0.9 \text{ mm}$



Figure 8 Winding product with a closed winding structure, gear ratio $u = \frac{491}{49}$

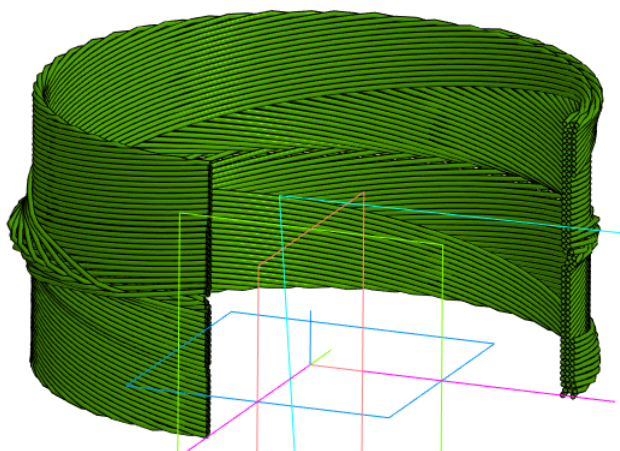


Figure 9 Computer model of thread winding with a closed structure, gear ratio $u = \frac{99}{49}$

Closed structure

$$u = \frac{494}{61} = 8 + \frac{6}{61} = 8 + \frac{1}{10 + \frac{1}{6}};$$

$$B = 250 \text{ mm}$$

$$D = 95 \text{ mm}$$

$$H = 61.7409 \text{ mm}$$

$$H_1 = 6.0729 \text{ mm}$$

$$H_2 = 1.0121$$

$$\alpha = 0.4155$$

$$\text{Thread diameter } d = 1 \text{ mm}$$

Same structure, but with different gear ratios

$$u = \frac{128}{61} = 2 + \frac{6}{61} = 2 + \frac{1}{10 + \frac{1}{6}};$$

$$B' = 64.7773 \text{ mm}$$

$$D = 95 \text{ mm}$$

$$H = 61.7409 \text{ mm}$$

$$H_1 = 6.0729 \text{ mm}$$

$$H_2 = 1.0121$$

$$\alpha = 0.4155$$

$$\text{Thread diameter } d = 1 \text{ mm}$$



Figure 10 Winding product with a closed winding structure, gear ratio $u = \frac{494}{61}$

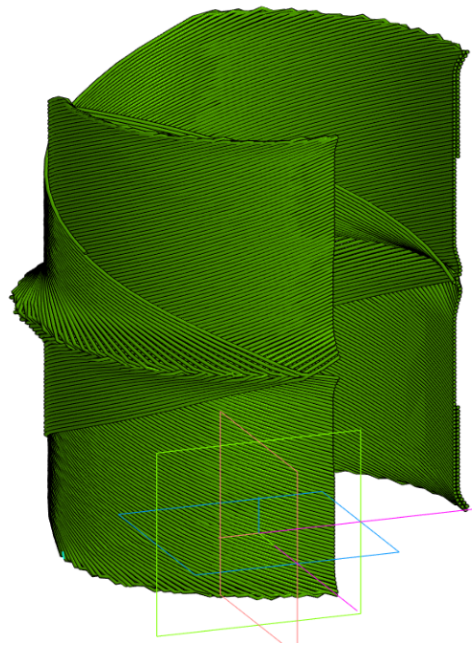


Figure 11 9 Computer model of thread winding with a closed structure, gear ratio $u = \frac{128}{61}$

Honeycomb structure

$$u = \frac{46}{5} = 9 + \frac{1}{5};$$

$$B = 250 \text{ mm}$$

$$D = 95 \text{ mm}$$

$$H = 54.3478 \text{ mm}$$

$$H_1 = 10.8696 \text{ mm}$$

$$H_2 = 0$$

$$\alpha = 0.2124$$

$$\text{Thread diameter } d = 1 \text{ mm}$$

Same honeycomb structure, but with different gear ratios

$$u = \frac{11}{5} = 2 + \frac{1}{5};$$

$$B' = 59.3478 \text{ mm}$$

$$D = 95 \text{ mm}$$

$$H = 54.3478 \text{ mm}$$

$$H_1 = 10.8696 \text{ mm}$$

$$H_2 = 0$$

$$\alpha = 0.2124$$

$$\text{Thread diameter } d = 1 \text{ mm}$$



Figure 12 Winding product with a honeycomb structure, gear ratio $u = \frac{46}{5}$

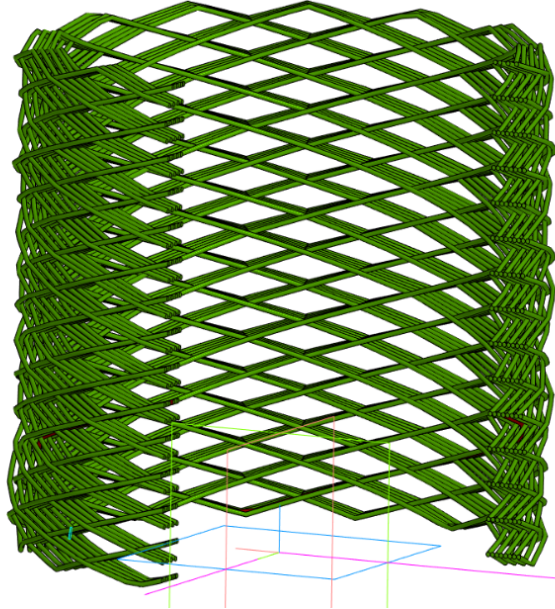


Figure 13 Computer model of thread winding with a honeycomb structure, gear ratio $u = \frac{11}{5}$

Honeycomb structure

$$u = \frac{68}{13} = 5 + \frac{3}{13} = 5 + \frac{1}{4 + \frac{1}{3}};$$

B = 250 mm

D = 95 mm

H = 95.5882 mm

H₁ = 22.0588 mm

H₂ = 7.3529 mm

α = 0.3260

Thread diameter d = 1 mm

Same honeycomb structure, but with different gear ratios

$$u = \frac{29}{13} = 2 + \frac{29}{13} = 2 + \frac{1}{4 + \frac{1}{3}};$$

B = 106.61765 mm

D = 95 mm

H = 95.5882 mm

H₁ = 22.0588 mm

H₂ = 7.3529 mm

α = 0.3260

Thread diameter d = 1 mm



Figure 14 Winding product with a honeycomb structure, gear ratio $u = \frac{68}{13}$

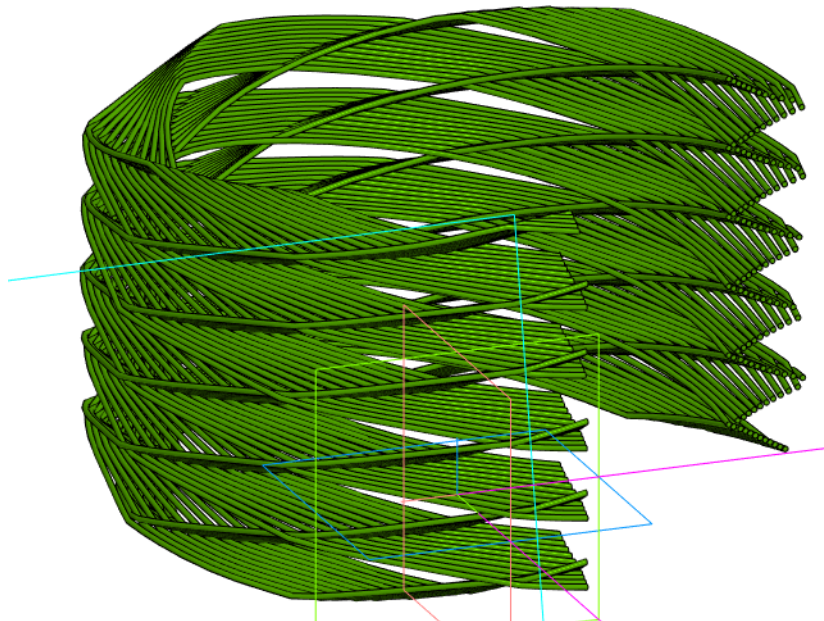


Figure 15 Computer model of thread winding with a honeycomb structure, gear ratio $u = \frac{29}{13}$

Spiral structure

$$u = \frac{992}{121} = 8 + \frac{24}{121} = 8 + \frac{1}{5 + \frac{1}{24}};$$

B = 250 mm

D = 95 mm

H = 60.9879 mm

H₁ = 12.0968 mm

H₂ = 0.5040 mm

α = 0.2124

Thread diameter d = 1 mm

Same structure, but with different gear ratios

$$u = \frac{266}{121} = 2 + \frac{24}{121} = 2 + \frac{1}{5 + \frac{1}{24}};$$

B' = 67.0363 mm

D = 95 mm

H = 60.9879 mm

H₁ = 12.0968 mm

H₂ = 0.5040 mm

α = 0.2124

Thread diameter d = 1 mm



Figure 16 Winding product with a spiral structure, gear ratio $u = \frac{992}{121}$

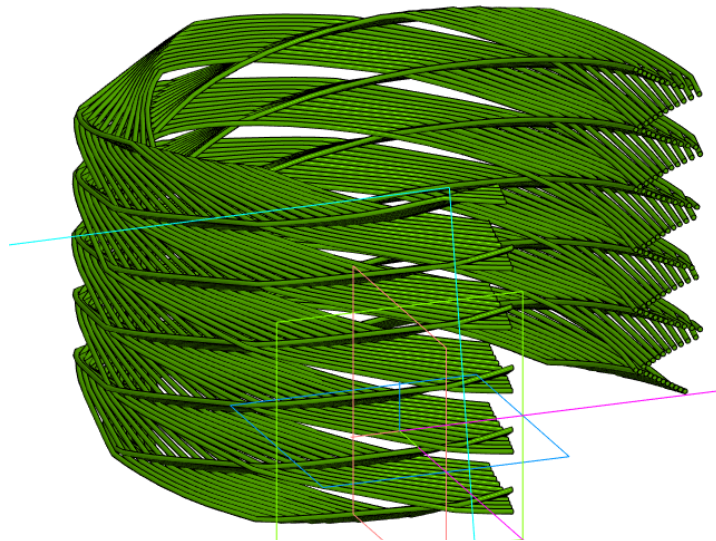


Figure 17 Computer model of thread winding with a spiral structure, gear ratio $u = \frac{266}{121}$

Spiral leading structure

$$u = \frac{1468}{179} = 8 + \frac{36}{179} = 8 + \frac{1}{5 + \frac{1}{8 + \frac{1}{11 + \frac{1}{1 + \frac{1}{2}}}}}$$

B = 250 mm

H=60.9673 mm

H₁ = 12.2616

H₂ = 0.3406

α = 0.2044

Thread diameter d = 1 mm

Same structure, but with different gear ratios

$$u = \frac{394}{179} = 2 + \frac{36}{179};$$

B' = 67.0981 mm

D = 95 mm

H = 60.9673 mm

H₁ = 12.2616

H₂ = 0.3406

α = 0.2044

Thread diameter d = 1 mm



Figure 18 Winding product with a spiral leading structure, gear ratio $u = \frac{1468}{179}$

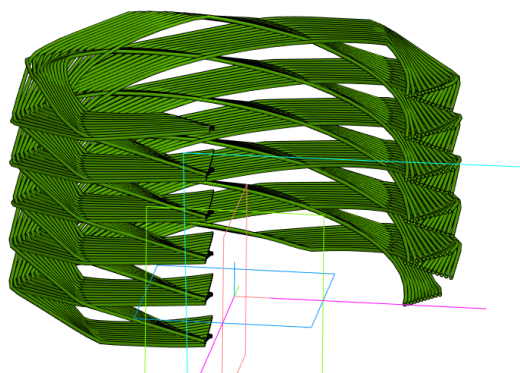


Figure 19 Computer model of thread winding with a spiral leading structure, gear ratio $u = \frac{394}{179}$

Let's summarize all the data in the table.

TABLE 2. STRUCTURE OF THE WP

No Experiment		B, mm	H, mm	H ₁ , mm	H ₂ , mm	α, рад	a ₀	a ₁	a ₂
1	a	250	124,7485	1,006	0	0,4155	4	124	0
	b	125,2516	124,7485	1,006	0	0,4155	2	124	0
2	a	250	49,8982	1,0183	0	0,1747	10	49	0
	b	50,4073	49,8982	1,0183	0	0,1747	2	49	0
3	a	250	61,7409	1,021	1,0121	0,4155	8	10	6
	b	64,7773	61,7409	1,021	1,0121	0,4155	2	10	6
4	a	250	54,3478	10,8696	0	0,2124	9	5	0
	b	59,3478	54,3478	10,8696	0	0,2124	2	5	0
5	a	250	95,5882	22,0588	7,3529	0,3260	5	4	3
	b	106,61765	95,5882	22,0588	7,3529	0,3260	2	4	3
6	a	250	60,9879	12,0968	0,5040	0,2124	8	5	24
	b	67,0363	60,9879	12,0968	0,5040	0,2124	2	5	24
7	a	250	60,9673	12,2616	0,3406	0,2044	8	5	8
	b	67,0981	60,9673	12,2616	0,3406	0,2044	2	5	8

5. Conclusions

Thus, the proposed method for developing computer 3D models of cylindrical winding products includes the following steps: creating a part file with a template; creating a 3D sketch of the winding turns (3D spiral) by importing point coordinates from an Excel file; creating auxiliary geometry, which uses the cross-sectional plane of the winding product passing through the axis of rotation and the starting point of the winding turns, positioned perpendicular to the trajectory; constructing a circle with a diameter d in the created plane; generating a 3D model of the thread using the "Sweep along path" operation. If processing a significant amount of data is required, these steps should be repeated. To reduce the data volume, a 3D model of only the repeating elements of the winding product should be created instead of the entire product. The winding is modeled as a continued fraction, and the repeating elements, which are shown on the unrolled winding path, are identified and analyzed in terms of geometry. 3D models of different types of windings are presented. Calculations of winding products with predefined parameters are provided. Repeating elements of the winding product structure are highlighted. This allows for the creation of an optimal 3D model that replicates the structure of any type of winding. The model can be used for further research and employed as a digital model of the winding product.

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